

Anomalous isotope effect near a 2.5 Lifshitz transition in a multi-band multi-condensate superconductor made of a superlattice of stripes

Andrea Perali^{1,2}, Davide Innocenti³, Antonio Valletta⁴ and Antonio Bianconi^{2,5}

¹ School of Pharmacy, Physics Unit, University of Camerino, I-62032 Camerino, Italy

² Mediterranean Institute of Fundamental Physics, Via Appia Nuova 31, I-00040 Marino, Italy

³ CNR-SPIN and Dipartimento di Ingegneria Informatica Sistemi e Produzione, 'Tor Vergata' University of Rome, Via del Politecnico 1, I-00133 Roma, Italy

⁴ CNR-IMM, Sezione di Roma, Via del Fosso del Cavaliere 100, I-00133 Roma, Italy

⁵ Rome International Center for Materials Science Superstripes (RICMASS) Via dei Sabelli 119A, I-00185 Roma, Italy

E-mail: andrea.perali@unicam.it

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Abstract

The doping dependent isotope effect on the critical temperature (T_c) is calculated for multi-band multi-condensate superconductivity near a 2.5 Lifshitz transition. We consider a superlattice of quantum stripes with finite hopping between stripes near a 2.5 Lifshitz transition for the appearance of a new sub-band making a circular electron-like Fermi surface pocket. We describe a particular type of BEC (Bose–Einstein Condensate) to BCS (Bardeen–Cooper–Schrieffer condensate) crossover in multi-band/multi-condensate superconductivity at a metal-to-metal transition that is quite different from the standard BEC–BCS crossover at an insulator-to-metal transition. The results show that the isotope coefficient strongly deviates from the standard BCS value 0.5, when the chemical potential is tuned at the 2.5 Lifshitz transition for the metal-to-metal transition. The critical temperature T_c shows a minimum due to the Fano antiresonance in the superconducting gaps and the isotope coefficient diverges at the point where a BEC coexists with a BCS condensate. In contrast T_c reaches its maximum and the isotope coefficient vanishes at the crossover from a polaronic condensate to a BCS condensate in the newly appearing sub-band.

(Some figures may appear in colour only in the online journal)

1. Introduction

In this work we focus on the isotope effect in nanostructures forming superlattices, where multicomponent superconductivity appears because of sub-bands due to quantum size effects. The electron wavefunctions for each sub-band are obtained by solving the Schrödinger equation for a one-dimensional modulated potential barrier. The k -dependent and

energy dependent superconducting gaps are calculated using the k -dependent anisotropic Bardeen–Cooper–Schrieffer (BCS) multi-gap equations. The key ingredient to solve the problem near a band edge is that the BCS multi-gap equation is solved jointly with the density equation, according with the Leggett approach currently used in ultracold fermionic gases. This scenario has been inspired by new experiments in cuprates discussed below, but the model points at predicting

new results on synthetic heterostructures. Therefore, in the first part of the introduction (Experiments) we introduce the novel emerging experimental scenario for the complex nanostructure of cuprates, while in the second part of the introduction (Theory) we present our theoretical model pointing to predict isotope effect in metallic nanostructures.

1.1. Experiments

While for many years most of the mechanisms proposed for high-temperature superconductivity have assumed a homogeneous lattice, recently, new experimental results have shifted the theoretical research toward complex materials showing multi-band/multi-condensate superconductivity. Following the discovery by ‘quantum oscillations’ experiments of the presence of a small electron Fermi surface [1] in cuprates, the community has considered the possibility of multi-band/multi-condensate superconductivity in charge density wave metals or spin density wave metals. The fundamental theoretical problems in this new scenario are similar to the superconductivity in ultra-narrow materials, where multi-band superconductivity is generated by quantum size effects due to the material lattice structure. These new results have supported previous results regarding the short-range lattice structure in cuprates showing deviations from the simple average structure. Experimental fast and local structural methods, such as extended x-ray absorption fine structure (EXAFS) [2], and x-ray absorption near-edge structure (XANES) [3], applied to cuprates [4, 5], reported below T^* the appearance of an incommensurate modulation of local lattice distortions. This incommensurate lattice modulation of the CuO_2 plane [7] is related with the lattice misfit strain between layers [8, 9], measured by the contraction of the Cu–O bond distance from the equilibrium distance of 197 pm [10]. The lattice modulation was assigned to self-organization of pseudo-Jahn–Teller polarons above a critical value of the misfit strain [11, 12]. The 1D lattice modulation was proposed to induce a periodic potential of 1D potential barriers in the superconducting planes [16, 17]. A 1D lattice modulation has been found also in pnictides [13], and evidence is accumulating for a 2.5 Lifshitz transition associated with a vanishing Fermi surface in electron-doped iron chalcogenides [18, 19], in diborides [14] and electron-doped cuprates [20].

In these materials the microscopic theory is very complex due to strong electron correlations, d-wave gaps and structural inhomogeneity [21, 22] made of scale invariant networks of superconducting grains [23, 24]. This scenario has been recently observed by imaging lattice fluctuations using nano x-ray diffraction [25, 26, 6] in $\text{La}_2\text{CuO}_{4+y}$, where each superconducting grain of the network is tuned near the shape resonance in superconducting gaps [27]. Moreover, the Fermi energy in the Fermi surface pocket in $\text{YBa}_2\text{Cu}_3\text{O}_{6+y}$ [1] is of the order of 35–40 meV. Remarkably, this very small Fermi energy E_F is of the order of the maximum value of the superconducting gap Δ measured at low temperature below T_c ($\Delta/E_F \approx 1$) by angle-resolved photo-emission spectroscopy (ARPES) and scanning tunneling microscopy (STM). These

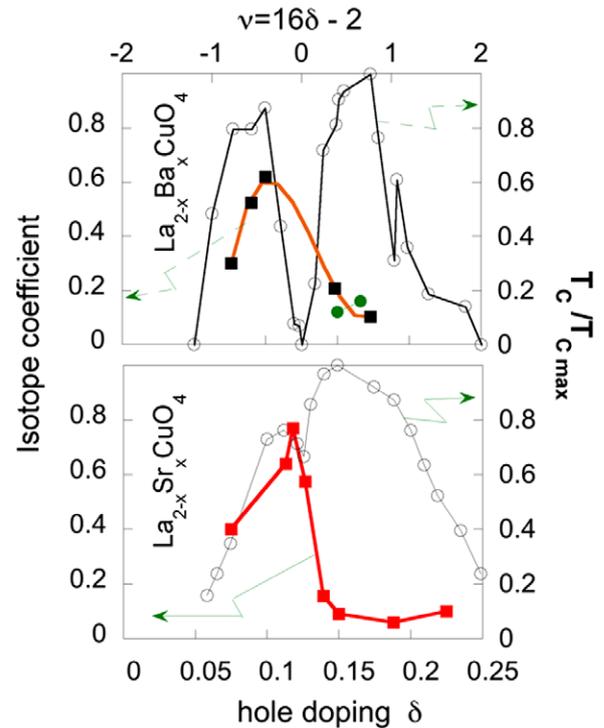


Figure 1. The doping dependent isotope coefficient α (filled squares) and the critical temperature (open circles) of $\text{La}_{2-x}\text{M}_x\text{CuO}_4$ ($M = \text{Sr}$ in the lower panel, $M = \text{Ba}$ in the upper panel) as a function of hole doping δ [30, 31]. The isotope coefficient for T_c is measured by oxygen isotope substitution, by replacing ^{16}O with ^{18}O , and shows a pronounced maximum for both compounds near doping $1/8$.

results suggest that the Fermi level is very close to a band edge of one of the multiple Fermi surfaces.

The isotope coefficient, α , of the superconducting critical temperature is close to 0.5 in conventional BCS superconductors [28] and independent of the shift of the chemical potential. This result depends on the assumption of a single Fermi surface where the Fermi level is far from the band edge and the attractive pairing mechanism is mediated by phonons. The experiments clearly show that this is not the case in cuprate high-temperature superconductors, where the isotope coefficient is doping dependent and nearly zero at optimum doping [29–36, 38–40]. This almost vanishing value of α has been considered key evidence for an unconventional non-phononic pairing mechanism in high-temperature superconductors [37, 41]. We consider the available experimental isotope effect in $\text{La}_{2-x}\text{M}_x\text{CuO}_4$ ($M = \text{Sr}, \text{Ba}$) [29–36, 38–40], since they show a single superconducting layer with the stripe phase at $1/8$. The isotope coefficient behavior in $\text{La}_{2-x}\text{M}_x\text{CuO}_4$ ($M = \text{Sr}, \text{Ba}$) has a very particular doping dependence [29–33], as shown in figure 1. It exhibits a strong anomaly, clearly shown near doping $1/8$ in figure 1, where the isotope coefficient peak reaches a value close to or larger than 0.5. In these $\text{La}_{2-x}\text{M}_x\text{CuO}_4$ systems the stripe phase is well established to appear at $1/8$ doping. The large isotope effect supports the involvement of the lattice dynamics in the pairing, but in a nontrivial way. The problem is very complex

since there are also effects of the isotope substitution on the electronic structure [42] and on the stripe phase [5]. The isotope coefficient α is expected to increase near the insulator-to-metal transition, where the polaron scenario is dominant [63], but the sharp anomaly at a particular doping indicates that the isotope coefficient shows a different anomaly related to the metallic stripe phase. The isotope effect has been discussed near a van Hove singularity in a single 2D band [43, 44] and in the frame of multi-band or multi-gap superconductivity far from band edges [45, 46], but there are theoretical efforts lacking on the investigation of the anomalous isotope coefficient in a multi-band superconductor near a band edge.

1.2. Theory

Inspired by the controversial discussions on this important topic by the scientific community, we focus in this paper on the generic features of the isotope effect in a multi-condensate superconductor near a band edge that can be tested in synthetic nanostructures and in the 2D electron gas at oxide interfaces. Multi-band multi-condensate superconductivity [47–49, 46] is usually considered for two coupled BCS condensates, where in the normal phase the Fermi energy is far from all band edges. The shape resonance in the superconducting gaps [47] is a type of Fano resonance between different pairing channels that occurs in multi-band metals where the chemical potential is driven at a 2.5 Lifshitz transition. The 2.5 Lifshitz transition in the proximity of a vanishing Fermi surface in a multi-band metal has been widely studied in the physics of Fermi surface topology in metals [50–53]. The shape resonances in superconducting gaps have been shown to occur in a single 2D ultra-thin metallic layer [54] and in a metallic stripe with 1D sub-bands or mini-bands [55, 56]. In 3D multilayer materials [57, 58, 14, 15], and in a superlattice of stripes [16, 17], the superlattice reduces quantum fluctuations in low dimensions and the high- T_c coherent phase can be realized. The T_c amplification is controlled by the Lifshitz energy parameter, measuring the energy difference between the chemical potential and the 2.5 Lifshitz transition. The maximum T_c is reached where the Lifshitz energy parameter is of the order of the energy cutoff for the pairing interaction [47, 14, 15].

In this work, we investigate a model of a superlattice of quantum stripes near a 2.5 Lifshitz transition. The fraction of the superconducting condensate originated by this small pocket has a quasi-bosonic character and is located in the crossover regime of the BCS–BEC (Bose–Einstein Condensation) crossover, a phenomenon which is intensively studied in ultracold fermions [59–61].

Therefore, our model can reproduce the formation of a quasi-bosonic condensate in the phase space where the small pocket appears. This type of BCS–BEC crossover is a generic feature of multi-band/multi-condensate superconductors when the chemical potential is tuned close to the bottom of one of the bands and the pairing is strong enough to open gaps of the order of the (small) Fermi energy.

While the standard BCS–BEC crossover [62, 63] has been studied in single-band metals, the present model provides a new scenario where the BCS–BEC crossover occurs in a multi-gap superconductor. Here, a first BCS condensate with order parameter Δ_1 in a large Fermi surface coexists with a second Bose-like condensate with order parameter Δ_2 in a second small electron pocket. Pair fluctuations and their screening in the multi-gap and multi-band (or multi-patch) models have been discussed also in the context of the physics of cuprates [64, 65].

In our model the Josephson-like coupling between the two condensates is a contact nonretarded interaction due to pair exchange mechanisms, which can be attractive or repulsive. The intraband pairing is mediated by an effective attraction, having the momentum and energy structure of the electron–phonon interaction in the BCS approximation. Note that in the case of electronic mechanisms, such as exchange of spin fluctuations (paramagnons), the repulsive interaction transforms in an attractive pairing thanks to the d-wave symmetry of the superconducting order parameter, because the characteristic (large) wavevector of the paramagnon connects states on the Fermi surface with opposite sign of the order parameter. Therefore both phononic and electronic mechanisms can be included in the model by an effective attractive interaction.

2. Model and methods

To investigate the response of the isotope effect at the Lifshitz transition, we consider the simplest physical model that grabs the essential physics of a superlattice of metallic stripes separated by a potential barrier [57, 4, 7, 11, 12, 55, 16] that makes a periodic potential in the 2D metallic layer,

$$W(y) = \sum_{n=-\infty}^{+\infty} W_b(y - nl_p), \quad (1)$$

where $W_b(y) = -V_b$ for $|y| \leq L/2$ and $W_b(y) = 0$ for $L/2 < |y| < l_p/2$, where L is the width of the confining well and l_p is the periodicity of the superlattice in the y direction.

The confining potential of equation (1) generates a band structure organized in mini-bands. This model allows us to simulate an electronic structure near the 2.5 Lifshitz transition for the appearance of a 2D FS. The Fermi surface topology changes by tuning the Fermi level E_F below the bottom of the second superlattice mini-band E_2 , where the superlattice FS (see figure 2) is made of a single FS of 1D character: the two open corrugated lines. The Fermi surface changes by tuning the Fermi level E_F above the bottom of the second superlattice sub-band E_2 , where a second closed FS of 2D character appears beyond the first 1D mini-band (see figure 2). This is determined by the quasi-free electron dispersion of the second sub-band $E_{2,k}^{2D} = E_2 + E(k_y) + k_x^2/2m$, where $E(k_y)$ is the energy dispersion in the y direction of the periodic potential $W(y)$ of the superlattice (here and in the following the reduced Planck constant is set to unity). The second closed Fermi surface changes its 2D topology into a 1D topology with isoenergetic open corrugated lines in the x direction,

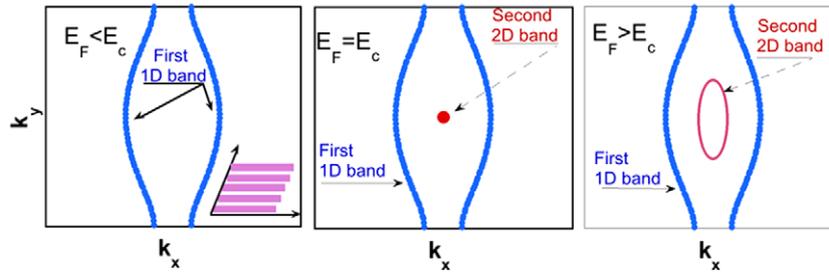


Figure 2. The evolution of the Fermi surface (FS) in our model for the striped phase that simulates a 2.5 Lifshitz phase transition (P_L) where a new detached electron-like Fermi surface appears. The inset of the left panel FS shows the lattice structure of a superlattice of stripes with finite hopping probability between stripes. E_c is the energy of the bottom of the second mini-band. The chemical potential is tuned from below (left panel $\delta < P_L$) to above E_c (central panel $\delta > P_L$), where a new small 2D closed electron Fermi surface appears (red circle). The right panel shows the FS beyond the electronic topological transition (ETT) of the second band from a 2D FS to a corrugated 1D FS at higher doping, called 2D–1D ETT.

above the energy threshold $E_{2D-1D} - E_2 = \xi$, where ξ is the energy band dispersion in the y direction and E_{2D-1D} is the energy where the topology of the FS changes from 2D to 1D. Therefore in our model we tune the Fermi level in the energy range $E_2 < E_F < E_{2D-1D}$. Multi-gap superconductivity in the energy range $E_2 - \xi < E_F < E_2 + \xi$ requires the theoretical approach we have recently proposed for a superlattice of superconducting layers [14, 15], which is capable of going beyond the standard BCS approximations, which consist of a single band, a large Fermi surface, a high Fermi energy, and a constant density of states (DOS) above and below E_F .

We consider a 1D periodic potential barrier of width B and wells of width L in the y direction with periodicity $l_p = L + B = 1.9$ nm and constant along the x direction. The potential barrier is fixed at $V_b = 1400$ meV. We note that small variations of l_p and of other parameters of the potential do not influence the main results of this work. The strength of the potential barrier V_b is important to determine the 1D–2D-dimensional crossover. The present choice of the periodic potential gives a band dispersion $\xi = 50$ meV. The band dispersion ξ is two times the electronic hopping integral t_y in the direction y , which is much smaller than the hopping integrals t_x in the x direction. Solving the Schrödinger equation for the 1D periodic potential barrier of equation (1), we obtain the wavefunctions of the electrons with a free electron band dispersion along the stripe direction and tight-binding mini-bands in the transverse direction. The eigenvalues are labeled by three quantum numbers $E = \epsilon_{n,k_x,k_y}$ where n is the mini-band index, and k_x and k_y are the components of the electron wavevectors in the superlattice. The DOS as function of the energy shows a jump at $E_{\text{edge}} = E_2$ and a sharp peak at E_{2D-1D} . The superconducting phase occurs because of the presence of attractive intraband electron–electron effective interactions (1, 1) and (2, 2) in the first and second band respectively and the interband exchange-like interactions (1, 2) and (2, 1). The cutoff energies of the interactions are symmetrically fixed around the Fermi surface and the value of the effective coupling has been fixed at $\lambda = 1/3$. In the BCS approximation, i.e., a separable interaction in wavevector space, the gap parameter has the same energy cutoff as the interaction. Therefore it has a value Δ_{n,k_y} around the Fermi surface in a range of energies equal

to the energy cutoff, depending on the mini-band index n and the superlattice wavevector k_y . The self-consistent equation for the ground-state ($T = 0$) energy gap Δ_{n,k_y} is:

$$\Delta_{n,k_y} = -\frac{1}{2N} \sum_{n',k'_x,k'_y} \frac{V_{\mathbf{k},\mathbf{k}'}^{n,n'} \Delta_{n',k'_y}}{\sqrt{(E_{n',k'_y} + \epsilon_{k'_x} - \mu)^2 + \Delta_{n',k'_y}^2}}, \quad (2)$$

where N is the total number of wavevectors in the discrete summation, μ is the chemical potential, $V_{\mathbf{k},\mathbf{k}'}^{n,n'}$ is the effective pairing interaction

$$V_{\mathbf{k},\mathbf{k}'}^{n,n'} = \tilde{V}_{\mathbf{k},\mathbf{k}'}^{n,n'} \theta(\omega_0 - |E_{n,k_y} + \epsilon_{k_x} - \mu|) \times \theta(\omega_0 - |E_{n',k'_y} + \epsilon_{k'_x} - \mu|) \quad (3)$$

calculated taking into account the interference effects between the wave functions of the pairing electrons in the different mini-bands, where n and n' are the mini-band indices, k_y (k'_y) is the superlattice wavevector and k_x (k'_x) is the component of the wavevector in the stripe direction of the initial (final) state in the pairing process, and

$$\tilde{V}_{\mathbf{k},\mathbf{k}'}^{n,n'} = -\frac{\lambda_{n,n'}}{N_0} S \times \int_S \psi_{n',-k'_y}(y) \psi_{n,-k_y}(y) \psi_{n,k_y}(y) \psi_{n',k'_y}(y) dx dy, \quad (4)$$

where N_0 is the DOS at E_F for a free electron 2D system, $\lambda_{n,n'}$ is the dimensionless coupling parameter, $S = L_x L_y$ is the surface of the plane and $\psi_{n,k_y}(y)$ are the eigenfunctions in the superlattice of quantum stripes. The gap equations have been solved iteratively. We obtain anisotropic gaps strongly dependent on the mini-band index and weakly dependent on the superlattice wavevector k_y . According with Leggett [62], the ground-state BCS wave function corresponds to an ensemble of overlapping Cooper pairs at weak coupling (BCS regime) and evolves to molecular (non-overlapping) pairs with bosonic character. The point is that the BCS equation for the gap has to be coupled to the equation that fixes the fermion density: with increasing coupling (or decreasing density), the chemical potential μ results as strongly renormalized with respect to the Fermi energy E_F of the noninteracting

system, and approaches minus half of the molecular-binding energy of the corresponding two-body problem in the vacuum. Therefore, in order to correctly describe the case of the chemical potential near a band edge, where all electrons in the newly appearing band condense, forming a bosonic-like gas in the second sub-band, the chemical potential in the superconducting phase is renormalized by the gaps opening at any chosen value of the charge density ρ :

$$\begin{aligned} \rho &= \frac{1}{L_x L_y} \sum_n \sum_{k_x, k_y} \left[1 - \frac{E_{n, k_y} + \epsilon_{k_x} - \mu}{\sqrt{(E_{n, k_y} + \epsilon_{k_x} - \mu)^2 + \Delta_{n, k_y}^2}} \right] \\ &= \frac{\delta k_y}{\pi} \sum_{n=1}^{N_b} \sum_{k_y=0}^{\pi/l_p} \int_0^{\epsilon_{\min}} d\epsilon \frac{2N(\epsilon)}{L_x} + \int_{\epsilon_{\min}}^{\epsilon_{\max}} d\epsilon \frac{N(\epsilon)}{L_x} \\ &\quad \times \left(1 - \frac{E_{n, k_y} + \epsilon_{k_x} - \mu}{\sqrt{(E_{n, k_y} + \epsilon_{k_x} - \mu)^2 + \Delta_{n, k_y}^2}} \right), \end{aligned} \quad (5)$$

where

$$\begin{aligned} \epsilon_{\min} &= \max[0, \mu - \omega_0 - E_{n, k_y}], \\ \epsilon_{\max} &= \max[0, \mu + \omega_0 - E_{n, k_y}], \\ N(\epsilon) &= \frac{L_x}{2\pi} \sqrt{\frac{\epsilon}{2m}}, \end{aligned}$$

and N_b is the number of the occupied mini-bands, L_x and L_y are the size of the considered surface and the increment in k_y is taken as $\delta k_y = 2\pi/L_y$. We compute the critical temperature T_c of the superconducting transition solving the linearized BCS equations

$$\Delta_{n, k_y} = -\frac{1}{2N} \sum_{n', k'} V_{\mathbf{k}, \mathbf{k}'}^{n, n'} \frac{\tanh\left(\frac{E_{n, k_y} + \epsilon_{k_x} - \mu}{2T_c}\right)}{E_{n, k_y} + \epsilon_{k_x} - \mu} \Delta_{n', k'}, \quad (6)$$

where the energy dispersion is measured with respect to the chemical potential. The iterations are stopped when a convergence factor of 10^{-6} has been reached, starting with a trial temperature T_1 and finding the T_c by the Newton tangent method to solve the implicit integral equation for T_c . The T_c is evaluated as a function of the chemical potential in the proximity of the edge of the second mini-band. The tuning of the chemical potential is measured by the Lifshitz parameter $z = (\mu - E_2)/\omega_0$, where E_2 is the bottom of the second band and ω_0 is the energy cutoff for the pairing interaction.

3. The isotope coefficient

The gaps, the T_c and the gap to T_c ratios have been plotted as a function of the Lifshitz energy parameter in the range $-1 < z < +1$, as shown in figure 3. We obtain the minimum of T_c , where μ is tuned in the BEC regime in the newly appearing circular Fermi surface in the range $-1 < z < 0$, depending on the strength of the Josephson-like coupling term. The maximum of T_c occurs where the topology of the FS of the second sub-band shows the 2D–1D electronic topological transition. This result is assigned to the shape resonance in the two superconducting gaps controlled by the

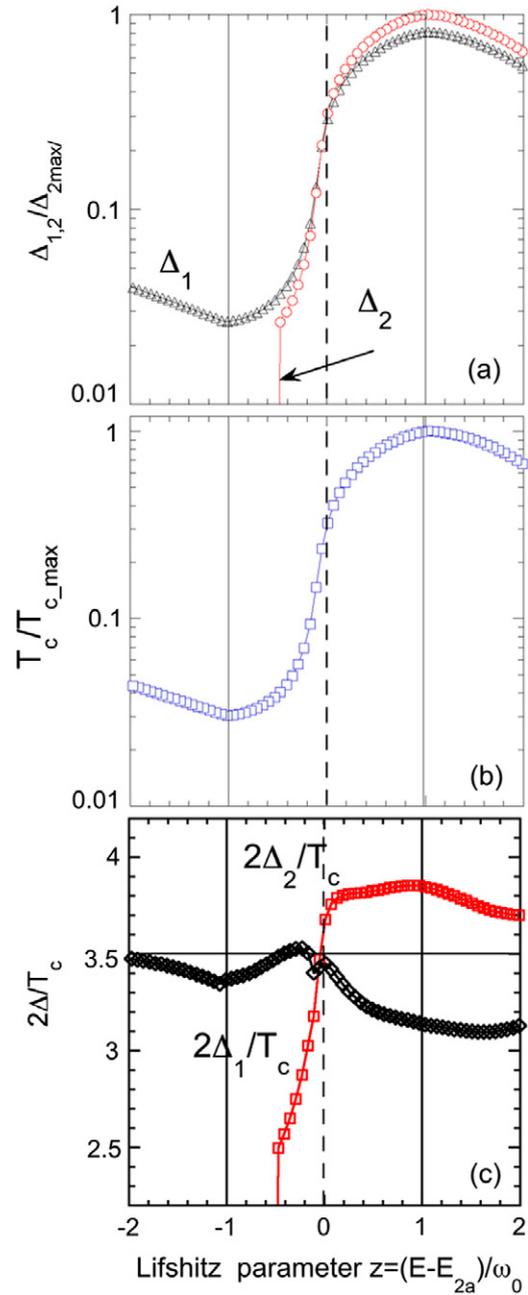


Figure 3. (a) The superconducting gaps in the first and second mini-bands normalized to the maximum value of the gap in the second mini-band; (b) the critical temperature T_c normalized to its maximum value; (c) and the gaps to T_c ratios for the two gaps as a function of the Lifshitz parameter $z = (\mu - E_2)/\omega_0$ compared with the standard BCS value of 3.5.

exchange-like pair-transfer (or Josephson-like) pairing, which shows a minimum (maximum) of T_c at the antiresonance at $z = -1$ (resonance at $z = 1$) due to the negative (positive) quantum interference effects typical of the shape resonances in the superconducting gaps.

The isotope coefficient $\alpha = \partial \ln T_c / \partial \ln M$ is calculated with the assumption of an energy cutoff of the interaction dependent on the isotopic mass as $\omega \propto M^{-1/2}$. In figure 4 we report the calculated isotopic coefficient α as a function of the chemical potential that strongly deviates from the standard

BCS value $\alpha = 0.5$. We find the maximum of T_c and the minimum of α at the shape resonance $\mu = E_{2D-1D}$, i.e., at $z = 1$ in figures 3 and 4. In contrast, by tuning the chemical potential at the 2.5 Lifshitz transition for the appearance of the second circular Fermi surface, i.e., at the band edge of the second mini-band $\mu = E_2$, we find a large value of $\alpha \gg 0.5$ and a drop of T_c , in agreement with the experimental data. The isotope coefficient has been calculated by considering the cases where only one of the intraband pairing energies $\omega_{11} \propto M^{-1/2}$ or $\omega_{22} \propto M^{-1/2}$ and the interband pairing energy is isotope dependent or independent. The best agreement with the experimental data reported in figure 4 is obtained for the case where both intraband pairing energy $\omega_{11} \propto M^{1/2}$ and $\omega_{22} \propto M^{1/2}$ are isotope dependent and the electronic interband energy is isotope independent. Note that, at the Lifshitz phase transition ($z = 0$), both α and the gap to T_c ratios in both bands get exactly the conventional BCS values. This is the crossing point between the Bose-like regime, where Δ_2 is smaller than Δ_1 but larger than $\mu - E_2$ (i.e., the Fermi surface in band 2 is destroyed by the gap opening), and the BCS-like regime, where Δ_2 is larger than Δ_1 but smaller than $\mu - E_2$ (i.e., the Fermi surface in band 2 is only partially smeared by the gap opening).

It is interesting that our results reproduce well the anomalous variation of the isotope coefficient and the critical temperature T_c versus doping in cuprates, as shown in figure 4.

4. Discussions and conclusions

We have obtained an interesting asymmetric feature of the chemical potential dependence of T_c and α in a superlattice of quantum wires tuned at a 2.5 Lifshitz topological transition. The energy cutoff ω_0 of the effective pairing interaction considered in our model determines the width of the shape resonance in the superconducting gaps and the small value of the isotope coefficient in the flat region of its doping dependence, well below the standard BCS value of $\alpha = 0.5$, where T_c has a maximum. We find the maximum T_c when the chemical potential is tuned near the 2D–1D electronic topological transition of the second sub-band (where the Lifshitz parameter assumes the value $z = 1$). The maximum value of the isotope coefficient is reached at the 2.5 Lifshitz transition, in the range of the Fano antiresonance, where T_c is strongly suppressed. Interestingly, the ratios between the gaps and T_c in different sub-bands cross the conventional BCS ratio value ($=3.5$) at the 2.5 Lifshitz transition, while sizable deviations from the conventional value are obtained in our calculations above this transition, in the range $1 < z < 2$ accessible to experiments. The present results have an impact on the physics of superconductivity in nano-sized superconductors [37, 41, 56], where the shape resonance in superconducting gaps is gaining momentum as a key ingredient for the road map for new high- T_c superconductors. Moreover, shape resonances and quantum size effects have been also considered in the context of superconductivity in nanofilms [66] and superfluidity in cigar-shaped ultracold Fermi gases [67] as a possible new driving mechanism to tune an atypical BCS–BEC crossover

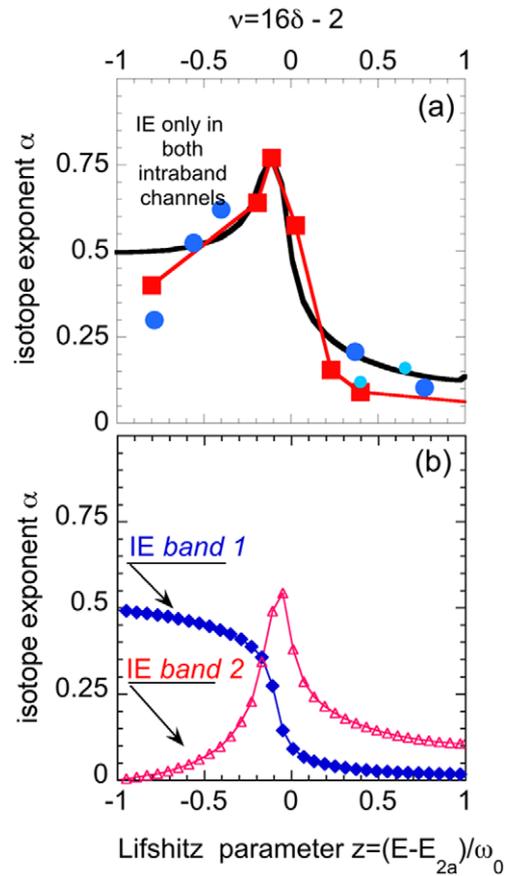


Figure 4. Panel (a) the experimental isotope coefficient α as a function of the reduced doping $\nu = 16\delta - 2$ for $\text{La}_{2-x}\text{M}_x\text{CuO}_4$ ($M = \text{Sr}$ dots and $M = \text{Ba}$ squares) compared with the calculated isotope coefficient α as a function of the Lifshitz parameter for the case of an isotope effect (IE) driven by intraband pairing in both bands $\omega_{11} \propto M^{1/2}$ and $\omega_{22} \propto M^{1/2}$, while the Josephson-like (exchange-like) pair-transfer mechanism is isotope independent; panel (b) calculated isotope coefficient α as a function of the Lifshitz parameter for the case of an isotope effect (IE) only in the intraband pairing energy in band 1 $\omega_{11} \propto M^{1/2}$ (filled dots) and for the case of an isotope effect (IE) only in the intraband pairing energy in band 2 $\omega_{22} \propto M^{1/2}$ (empty triangles).

in multi-band fermionic systems, with the appearance of a coherent mixture of BCS-like and BEC-like condensate. Finally, we propose that the anomalous maximum of the isotope effect and its peculiar doping dependence in the doping range $1/8$, shown in figure 1 in La_{214} families, could have its origin in a 2.5 Lifshitz transition for a metal-to-metal transition in a multi-band multi-condensate superconductor, for the appearance or disappearance of small electron-like Fermi surface pocket, in agreement with a large Nernst effect measured in the normal phase [51, 52]. Further work is needed, including in the theoretical model electron–electron correlations and superconducting gaps with d-wave symmetry, for a quantitative explanation of the anomalous isotope effect in superconducting cuprates. Our predictions can be tested by experiments of the isotope effects in superconducting nanostructures, tuning the chemical potential by gate voltage, and in iron chalcogenides

superconductors, driving the chemical potential by electron doping, around a 2.5 Lifshitz transition.

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